

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Thursday 12 June 2025 – Afternoon

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Time allowed: 2 hours

plus your additional time allowance

YOU MUST HAVE:

**the Printed Answer Booklet or any suitable paper
provided by the centre. The Printed Answer Booklet may
be enlarged by the centre.**

a scientific or graphical calculator

Insert for Question 9 (with this document)

Loose Sheet for Question 13 (with this document)

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.

The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 100.

The marks for each question are shown in brackets [].

ADVICE

Read each question carefully before you start your answer.

FORMULAE

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\}$$

Geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \quad \text{for } |r| < 1$$

Binomial series

$$(a + b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n$$

$(n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$

$$(|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x) (f(x))^n dx = \frac{1}{n+1} (f(x))^{n+1} + c$$

Integration by parts $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Small angle approximations

$\sin \theta \approx \theta$, $\cos \theta \approx 1 - \frac{1}{2}\theta^2$, $\tan \theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi \right)$$

Numerical methods

Trapezium rule:

$$\int_a^b y \, dx \approx \frac{1}{2}h \left\{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \right\},$$

$$\text{where } h = \frac{b - a}{n}$$

The Newton-Raphson iteration for solving $f(x) = 0$:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B)$$

$$\text{OR } P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{OR} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$,

mean of X is np , variance of X is $np(1 - p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995
z	0.674	1.282	1.645	1.960	2.326	2.576

p	0.9975	0.999	0.9995
z	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

SECTION A

Pure Mathematics

1 In this question you must show detailed reasoning.

Solve the inequality $x^2 + 3x \leq 10$. [3]

2 In this question you must show detailed reasoning.

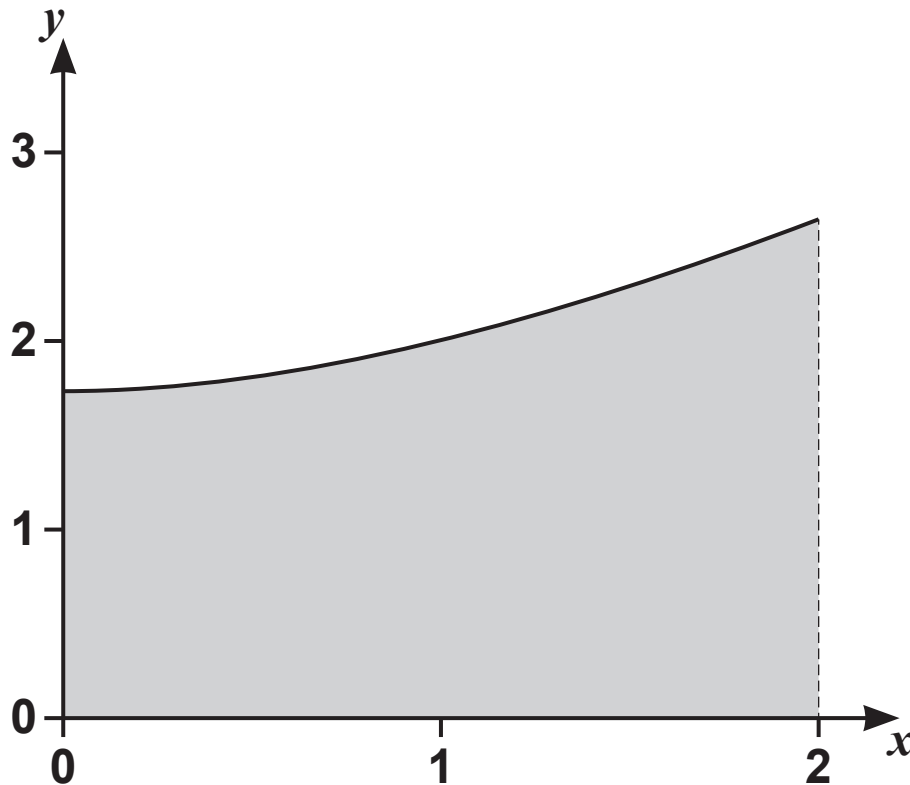
Solve the following equations.

(a) $(x^2 - 5)^{\frac{3}{2}} = 8$ [3]

(b) $e^{3y} = 2$ [2]

(c) $x^4 - 3x^2 - 4 = 0$ [4]

- 3 The diagram shows part of the curve with equation $y = \sqrt{x^2 + 3}$, between $x = 0$ and $x = 2$.**



- (a) (i) Use the trapezium rule, with intervals of 0.5, to determine an approximation to the area of the shaded region. [3]**
- (ii) Use rectangles with the same intervals as in part (i) to determine lower and upper bounds for the exact area of the shaded region. [2]**
- (b) Write an expression for the exact area of the shaded region in terms of a limit, involving x and intervals of width δx . [2]**

4 (a) Prove that $(\cos \theta + \sin \theta)^2 \equiv 1 + \sin 2\theta$. [2]

(b) Hence or otherwise prove that

$$\sec 2\theta + \tan 2\theta \equiv \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}. \quad [4]$$

5 In this question you must show detailed reasoning.

A circle has diameter PQ where P is $(-5, 1)$ and Q is $(5, 1)$. The line $x + 2y = 12$ meets the circle at A and B .

Find the exact length AB . [8]

6 A curve has equation $y = 3x^4 - 4x^3 - 6x^2 + 12x$.

(a) (i) Find $\frac{dy}{dx}$. [2]

(ii) Use your answer to part (a)(i) to find the x -coordinates of the stationary points on the curve. [2]

(b) Show that there is exactly ONE point on the curve which is a point of inflection but not a stationary point. [3]

- 7** An arithmetic progression has first term a and common difference d , where a and d are non-zero. The first, third and fourth terms of the arithmetic progression are consecutive terms of a geometric progression with common ratio r .

(a) (i) Show that $r = \frac{a + 2d}{a}$. [1]

(ii) Find d in terms of a . [2]

(b) Find the common ratio of the geometric progression. [2]

- 8** In this question you must show detailed reasoning.

Use integration by parts to find the exact value of

$$\int_0^{\frac{1}{2}} \ln(3 - 2x) \, dx. \quad [5]$$

SECTION B

Statistics

- 9** Some students in a year-group took two tests, Test 1 and Test 2. The maximum mark in each test was 120. The students' marks are illustrated in the cumulative frequency diagram. The diagram is reproduced in the insert.

- (a)** All the students in the year-group took Test 1, but some students missed Test 2.

State the number of students who missed Test 2.

[1]

- (b)** Find an estimate of the number of students whose marks for Test 1 were in the range 45 to 65 inclusive. **[2]**

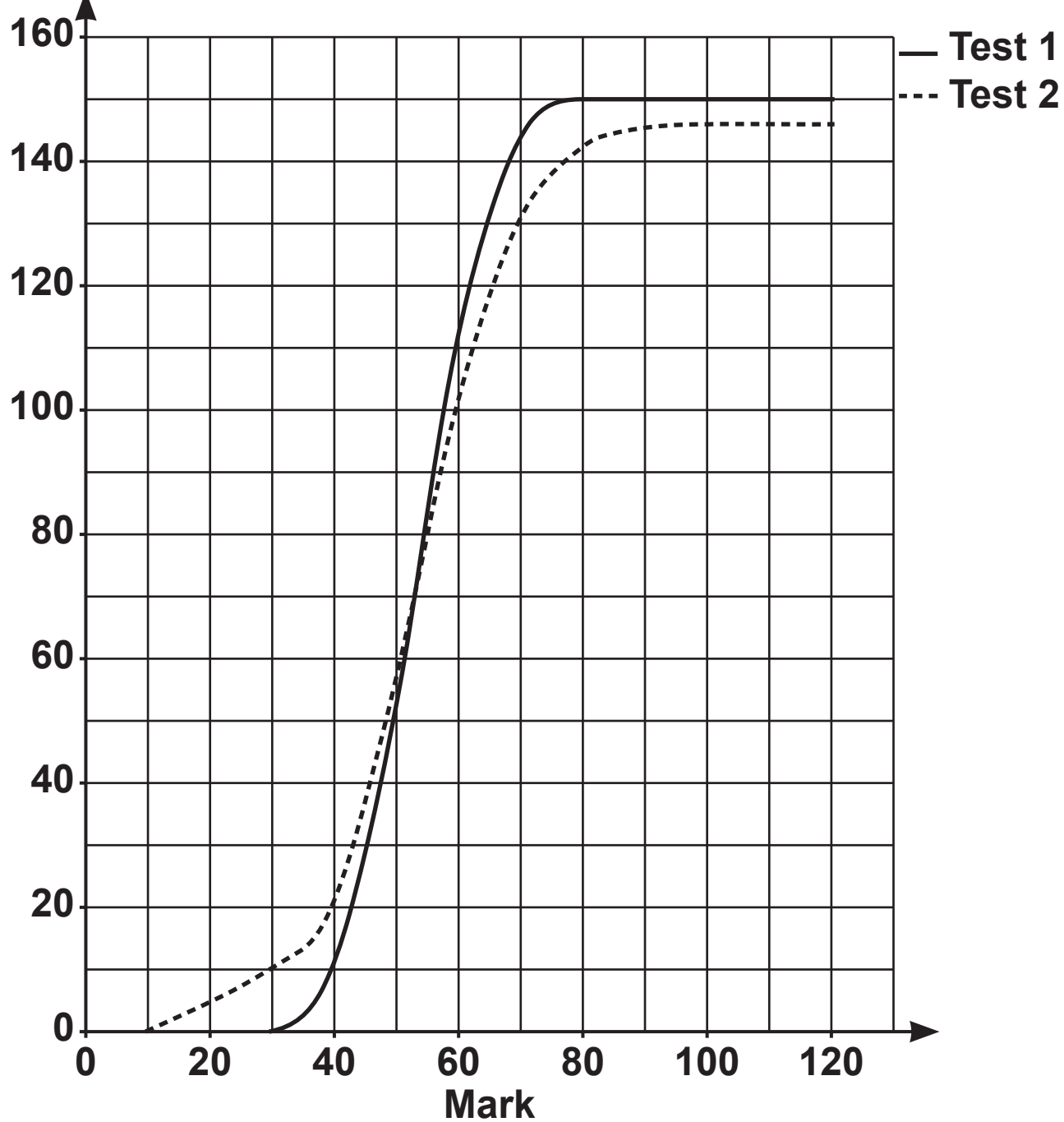
- (c)** The final parts of the graphs are horizontal.

For Test 1, explain what this means about the results. [1]

- (d)** It is required that the number of students obtaining grade A* should be the same on both tests. The minimum mark for grade A* on Test 2 was 76.

Find the minimum mark for grade A* on Test 1. [1]

Cumulative
frequency



A teacher commented that the median marks on both tests were very similar. This suggests that, on average, the levels of difficulty on both tests were about the same.

- (e) Use the cumulative frequency diagram to make another comparison between Test 1 and Test 2 about the easier questions on both papers. [1]**

The marks for Test 1 are summarised in the table.

Test 1 mark	≤ 29	30–39	40–49	50–59	60–69	70–79	≥ 80
Frequency	0	10	40	60	33	7	0

- (f) (i) Find estimates of the mean and standard deviation of the Test 1 marks. [3]**
- (ii) Use your answers to part (f)(i) to comment on whether there may be any outliers amongst the Test 1 marks. [2]**

- 10 (a) The random variable W has the distribution $B(12, \frac{1}{3})$.

Find $P(W \geq 7)$. [2]

- (b) The random variable X has the distribution $N(45, 4^2)$.

Find $P(X > 50)$. [1]

- (c) The random variable Y has the distribution $N(25, \sigma^2)$.

Given that $P(Y < 30) = 0.75$, find σ . [2]

- 11 A train company claims that a particular daily service arrives on time on 95% of days. Riley suspects that the true percentage is less than 95%. Riley tests the company's claim by choosing a random sample of 50 days during the period November 2023 to March 2024. Riley finds that the service arrived on time on 44 of these days.

- (a) Use a binomial distribution to test at the 5% significance level whether Riley's suspicion is justified. [7]
- (b) By referring to one of the necessary assumptions for using a binomial distribution, suggest a reason why the conclusion may NOT be reliable. [1]

- 12 Sam has 9 cards, each with a different non-zero digit printed on it.**



Sam chooses 5 cards at random and places them in a random order in a straight line, to form a 5-digit number.

- (a) (i) Find the probability that the 5-digit number is greater than 50 000. [1]**
- (ii) Show that the probability that the 5-digit number is greater than 58 600 is $\frac{13}{28}$. [4]**
- (b) Given that the 5-digit number is greater than 50 000, determine the probability that it is greater than 58 600. [2]**

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- 13 The table on the Loose Sheet shows excerpts from the census data for Age Structure in 2001 and 2011 for four Local Authorities (LAs) in South West England.**

Researchers want to investigate whether there is evidence that those people who were residents in these LAs in 2001 were still resident in the same LAs in 2011.

- (a) One researcher suggests that, for each of the LAs, they should compare the sum of the data for 2001 for Age 15 to 19 with the data for 2011 for Age 25 to 29.**

Explain why this comparison is relevant. [1]

- (b) The census data also includes data for the following age ranges:**

Age						
30 to 44	45 to 59	60 to 64	65 to 74	75 to 84	85 to 89	90 +

Explain why NONE of these data ranges are helpful in this context. [1]

- (c) The data for one of these four LAs show that some of the 0 to 4 year olds living in that LA in 2001 are definitely no longer living there in 2011.**

Explain which LA this is. [1]

(d) One of the researchers says that there has been little movement in or out of Bournemouth between 2001 and 2011 for those who were aged 5 to 7 in 2001.

(i) Use values from the table to show that this statement is NOT contradicted by the data. [2]

(ii) Explain why the statement is NOT necessarily correct. [1]

14 The masses of stones on a certain beach are normally distributed with mean 0.36 kg and standard deviation 0.12 kg.

(a) Find the probability that two stones chosen at random from this beach both have masses between 0.3 kg and 0.4 kg. [2]

(b) Determine the probability that the mean mass of a random sample of 40 stones from this beach is less than 0.32 kg. [2]

Some geologists suspect that the mean mass, μ kg, of stones on a second beach is different from the mean for the first beach. They carry out a hypothesis test, at the 5% significance level, of the null hypothesis $\mu = 0.36$ against the alternative hypothesis $\mu \neq 0.36$.

They weigh each of a random sample of 40 stones on the second beach and find the sample mean, $\bar{X}$ kg, of their masses. You may assume that the standard deviation of the masses of stones on the second beach is 0.12 kg.

(c) Determine the range of values of $\bar{X}$ for which the null hypothesis would NOT be rejected. [5]

15 The probability distribution of the random variable X is modelled as follows.

$P(X = 1) = p$ where p is a constant.

$P(X = x) = 2P(X = x - 1)$ for $x = 2, 3, 4$.

$P(X = x) = 0$ for all other values of x .

Three values, X_1 , X_2 and X_3 , of X are chosen at random.

Determine $P(X_3 > X_1 + X_2)$. [4]

END OF QUESTION PAPER

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